Please denote your answers clearly, i.e., box in, star, etc., and write neatly. There are no points for small, messy, unreadable work... please use lots of paper.

Problem 1: Hibbeler, 12–49.
The $v - t$ graph for the motion of a car as it moves along a straight road is shown. Draw the $a - t$ graph and determine the maximum acceleration during the 30 s time interval. The car starts from rest at $x = 0$.

Solution:
The acceleration is the derivative of velocity, so that $a = \dot{v}$. Therefore the acceleration takes two forms, the first during the interval $0 \leq t \leq 10$, for which

$$\ddot{x}(t) = \dot{v}(t) = (0.8 \text{ ft/s}^3) t.$$  

During the second interval the acceleration reduces to

$$\ddot{x}(t) = \dot{v}(t) = 1.00 \text{ ft/s}^2.$$  

The $a - t$ graph is shown to the right. Finally, the maximum acceleration occurs at $t = 10$ s, for which $\ddot{x}(t) = 8.00 \text{ ft/s}^2$.

Problem 2: Hibbeler, 12–81.
Show that if a projectile is fired at an angle $\theta$ from the horizontal with an initial velocity $v_0$, the maximum range the projectile can travel is given by $R_{\text{max}} = \frac{v_0^2}{g}$, where $g$ is the acceleration of gravity.

What is the angle $\theta$ for this condition?

Solution:
The position of the ball is described by $r_{PO} = x(t) \hat{i} + y(t) \hat{j}$. When subject to gravity, and initial velocity,

$$\mathbf{\mathcal{F}_{P}} = \dot{x}(0) \hat{i} + \dot{y}(0) \hat{j} = v_0 (\cos \theta \hat{i} + \sin \theta \hat{j}),$$

the equations for the motion of the particle are written as
\[ x(t) = (v_0 \cos \theta) t, \quad y(t) = -\frac{g t^2}{2} + (v_0 \sin \theta) t. \]

Therefore, when \( y \) is written as a function of \( x \), this reduces to

\[
y(x) = -\frac{g}{2} \frac{\sin \theta}{\cos \theta} x^2 + \sin \theta \cos \theta x = \frac{g x}{v_0^2 (1 + \cos(2\theta))} \left( x - \frac{v_0^2 \sin(2\theta)}{g} \right).
\]

The range occurs when \( y = 0 \), or solving for \( x \)

\[
R = \frac{v_0^2 \sin(2\theta)}{g}.
\]

The maximum range thus occurs for \( 2\theta = 90^\circ \), or

\[
\theta = 45^\circ, \quad \rightarrow \quad R_{\text{max}} = \frac{v_0^2}{g}.
\]

**Problem 3:** Hibbeler, 12–90.

The fireman standing on the ladder directs the flow of water from his hose to the fire at \( B \). Determine the velocity of the water at \( A \) if it observed that the hose is held at \( \theta = 20^\circ \).

**Solution:**

The vertical displacement of the water \( y \) as a function of the horizontal displacement \( x \) is given as

\[ y(x) = -\frac{g}{2} \frac{\sin \theta}{\cos \theta} x^2 + \sin \theta \cos \theta x. \]

The initial angle is given as \( \theta = -20^\circ \), while the final displacement is \((x_f, y_f) = (60 \text{ ft}, -30 \text{ ft})\). Therefore, solving for \( v_0 \) yields

\[ v_0 = \sqrt{\frac{g}{2 \cos \theta (\sin \theta x_f - \cos \theta y_f)}} x_f. \]

With the given values, this reduces to \( v_0 = 89.68 \text{ ft/s} \).
Problem 4: Hibbeler, 12–94.
The stones are thrown off the conveyor with a horizontal velocity of 10 ft/s as shown. Determine the speed at which the stones hit the ground at $B$.

Solution:
The position of a stone can be described with the vector $\mathbf{r}_{p_0} = x\mathbf{i} + y\mathbf{j}$, so that using $x(t) = (v_0 \cos \theta)t$, $y$ can be written as a function of $x$ as

$$y(x) = -\frac{g}{2v_0^2 \cos^2 \theta} x^2 + \frac{\sin \theta}{\cos \theta} x.$$  

For this system $v_0 = 10$ ft/s, and $\theta = 0^\circ$, which reduces the above to

$$y(x) = -\frac{g}{2v_0^2} x^2.$$  

The ground at the bottom of the conveyor can be described with the equation

$$y_{gr} = -(100 \text{ ft}) - \frac{x_{gr}}{10}.$$  

Therefore, the stones hit the ground when their trajectory intersects the equation for the surface. That is

$$-\frac{g}{2v_0^2} x_f^2 = y_f = -(100 \text{ ft}) - \frac{x_f}{10} \quad \rightarrow \quad (0.161 \text{ ft}^{-1}) x_f^2 - 0.10 x_f - (100 \text{ ft}) = 0.$$  

This equation is quadratic in $x_f$, and may be solved to yield $x_f = 25.23$ ft. To find the speed at which the stones hit the ground, we return to the equations for the velocity, which can be written as

$$\dot{x}(t) = v_0, \quad \dot{y}(t) = -g t = -\frac{g}{v_0} x.$$  

Finally, the speed of the stones at impact can be written as

$$\|\mathbf{v}_p\| = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{v_0^2 + \left(\frac{g x_f}{v_0}\right)^2}.$$  

Using the above value $x_f = 25.23$ ft, we find that the stones hit the ground with speed $\|\mathbf{v}_p\| = 81.87 \text{ ft/s}$.

Problem 5: Hibbeler, 12–145.
A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a speed of 20 m/s which is increasing at 3 m/s$^2$. Determine the truck’s radial and transverse components of acceleration.

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Problem 5: Hibbeler, 12–145.
A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a speed of 20 m/s which is increasing at 3 m/s$^2$. Determine the truck’s radial and transverse components of acceleration.
Solution:

Given the path of the truck, it is natural to describe its position in terms of polar coordinates, so that

\[
\begin{align*}
\mathbf{r}_{PO} &= r \hat{e}_r, \\
\mathbf{v}_P &= \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta, \\
\mathbf{a}_P &= \left(\ddot{r} - r \dot{\theta}^2\right) \hat{e}_r + \left(r \ddot{\theta} + 2 \dot{r} \dot{\theta}\right) \hat{e}_\theta.
\end{align*}
\]

With constant radius of the curve, \( \dot{r} = 0 \) and \( \ddot{r} = 0 \), so that the kinematics reduce to

\[
\begin{align*}
\mathbf{r}_{PO} &= r \hat{e}_r, \\
\mathbf{v}_P &= \left(r \dot{\theta}\right) \hat{e}_\theta, \\
\mathbf{a}_P &= \left(-r \dot{\theta}^2\right) \hat{e}_r + \left(r \ddot{\theta}\right) \hat{e}_\theta.
\end{align*}
\]

The speed of the truck, 20 m/s, is given in terms of the coordinates as

\[
\|\mathbf{v}_P\| = r \dot{\theta} = 20 \text{ m/s},
\]

while its rate of change is

\[
\frac{d}{dt} (\|\mathbf{v}_P\|) = r \ddot{\theta} = 3 \text{ m/s}^2.
\]

Notice that the rate of change of the speed is different from the magnitude of the acceleration. From these, we can determine \( \dot{\theta} \) and \( \ddot{\theta} \) as

\[
\begin{align*}
\dot{\theta} &= \frac{1}{3} \text{ rad/s}, \\
\ddot{\theta} &= \frac{1}{20} \text{ rad/s}^2.
\end{align*}
\]

Finally, with these values the acceleration of the truck can be written as

\[
\begin{align*}
\mathbf{a}_P &= \left(-r \dot{\theta}^2\right) \hat{e}_r + \left(r \ddot{\theta}\right) \hat{e}_\theta = \left(-\frac{20}{3} \text{ m/s}^2\right) \hat{e}_r + \left(3 \text{ m/s}^2\right) \hat{e}_\theta.
\end{align*}
\]
Problem 6: Hibbeler, 12–152.

At the instant shown, the water sprinkler is rotating with an angular speed \( \dot{\theta} = 2 \, \text{rad/s} \) and an angular acceleration \( \ddot{\theta} = 3 \, \text{rad/s}^2 \). If the nozzle lies in the vertical plane and water is flowing through it at a constant rate of 3 m/s

a) determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, \( r = 0.2 \, \text{m} \);

b) (this part is not in the textbook but builds upon this problem) once it exits the nozzle, find how far this water particle travels before hitting the ground. Assume that the nozzle is at ground level.

Solution:

a) The kinematics of a particle of water \( P \) can be described in terms of polar coordinates as

\[
\begin{align*}
\mathbf{r}_{PO} &= r \hat{e}_r, \\
\dot{\mathbf{r}} &= \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta, \\
\ddot{\mathbf{r}} &= \left( \ddot{r} - r \dot{\theta}^2 \right) \hat{e}_r + \left( r \ddot{\theta} + 2 \dot{r} \dot{\theta} \right) \hat{e}_\theta.
\end{align*}
\]

From the problem statement, the coordinates and their derivatives of \( P \) at the nozzle exit are given as

\[
\begin{align*}
\dot{r} &= 3 \, \text{m/s}, \\
\ddot{r} &= 0, \\
\dot{\theta} &= 2 \, \text{rad/s}, \\
\ddot{\theta} &= 3 \, \text{rad/s}^2.
\end{align*}
\]

Notice that the coordinate \( \theta \) is not given, and does not influence the kinematics when written in terms of the radial and tangential directions (\( \theta \) certainly does affect the orientation of \( \hat{e}_r \) and \( \hat{e}_\theta \) relative to the ground). The velocity and acceleration can be written as

\[
\begin{align*}
\dot{\mathbf{r}} &= \left( 3 \, \text{m/s} \right) \hat{e}_r + \left( 0.4 \, \text{m/s} \right) \hat{e}_\theta, \\
\ddot{\mathbf{r}} &= \left( -0.8 \, \text{m/s}^2 \right) \hat{e}_r + \left( 12.6 \, \text{m/s}^2 \right) \hat{e}_\theta.
\end{align*}
\]

Finally, the magnitudes of these quantities are

\[
\left\| \dot{\mathbf{r}} \right\| = 3.03 \, \text{m/s} \quad \text{and} \quad \left\| \ddot{\mathbf{r}} \right\| = 12.6 \, \text{m/s}^2.
\]

Notice that the velocity of the water is not simply in the \( \hat{e}_r \) direction. Instead, the water velocity is directed at an angle of 7.59° off the \( \hat{e}_r \) direction.
b) The range of the water can be determined from the equation
\[ R = \frac{v_0^2 \sin(2\psi)}{g}, \]
where the water has an exit speed of \( v_0 \) and a velocity direction of \( \psi \). Using the above values, we find that \( v_0 = 3.03 \text{ m/s} \) and \( \psi = \theta + 0.13 \text{ rad} \), so that
\[ R = (0.93 \text{ m}) \sin (2(\theta + 0.13 \text{ rad})). \]

In contrast, if the nozzle is held stationary at an angle \( \theta \), the range of the sprinkler is
\[ R = (0.92 \text{ m}) \sin (2\theta). \]

Problem 7: Hibbeler, 12–154.

A cameraman standing at \( A \) is following the movement of a race car, \( B \), which is traveling along a straight track at a constant speed of 80 ft/s. Determine the angular rate at which he must turn in order to keep the camera directed on the car at the instant \( \theta = 60^\circ \).

Solution:

With the perpendicular distance between the track and \( O \) given as \( d \), the distance \( r \) between \( O \) and \( C \) is
\[ ||r_{CO}|| = r = \frac{d}{\sin \theta}. \]

Here the velocity of the car is naturally written in terms of Cartesian coordinates. However, the response of the cameraman is determined in terms of polar coordinates. In terms of the former, the velocity of the care is written as
\[ \mathbf{v}_C = -v_0 \mathbf{i}, \]
while in terms of the polar coordinates \( r \) and \( \theta \)
\[ \mathbf{v}_C = \dot{r} \mathbf{e}_r + (r \dot{\theta}) \mathbf{e}_\theta, \]
with
\[ \dot{\mathbf{e}}_r = \cos \theta \mathbf{i} + \sin \theta \mathbf{j} \quad \dot{\mathbf{e}}_\theta = -\sin \theta \mathbf{i} + \cos \theta \mathbf{j} \]
\[ \dot{i} = \cos \theta \dot{e}_r - \sin \theta \dot{e}_\theta, \quad \dot{j} = \sin \theta \dot{e}_r + \cos \theta \dot{e}_\theta. \]

Setting these two descriptions of \( \mathbf{v}_C \) equal to one another, we find that
\[ -v_0 \mathbf{i} = \dot{r} \mathbf{e}_r + (r \dot{\theta}) \mathbf{e}_\theta. \]
This vector equation has two unknowns, \( \dot{r} \) and \( \dot{\theta} \). We could write the directions \( \mathbf{e}_r \) and \( \mathbf{e}_\theta \) in terms on \( \mathbf{i} \) and \( \mathbf{j} \), which would lead to two scalar equations, coupled in the two
unknowns. However, instead we write $i$ in terms of $\hat{e}_r$ and $\hat{e}_\theta$. Doing so yields

$$-v_0 (\cos \theta \hat{e}_r - \sin \theta \hat{e}_\theta) = \dot{r} \hat{e}_r + (r \dot{\theta}) \hat{e}_\theta.$$  

Thus, solving the equation in the $\hat{e}_\theta$ for $\dot{\theta}$ provides

$$\dot{\theta} = \frac{v_0 \sin \theta}{r} = \frac{v_0 \sin^2 \theta}{d}.$$  

Therefore, at $\theta = 60^\circ$, we find that $\dot{\theta} = 0.6 \text{ rad/s}$. 